# CONTINUUM SOLUTIONS FOR DRAPING AND DIFFERENTIAL COMPACTION OF COMPRESSIBLE ELASTIC LAYERS— IMPLICATIONS FOR THE ORIGIN AND GROWTH OF EARTH FISSURES

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In order to investigate the relation between draping of compressible surficial layers over buried irregularities and zones of ground failure (Holzer, 1984; Helm, 1992), continuum solutions have been developed to model the deformation both of single- and multiple-layered elastic bodies subjected to vertical displacement of the lower boundary (Haneberg, 1992, 1993). The geometry and boundary conditions for a single layer model are illustrated in figure 1. These solutions are limited to originally flat, homogeneous, and isotropic elastic layers subjected to lower boundary displacements that can be specified using a Fourier sine or cosine series. The upper surface of the layer(s) is traction-free, corresponding to the Earth's surface. Although the new solutions are not as versatile as finite element solutions previously used to model deformation associated with earth fissures, for example by Jachens and Holzer (1979) and Larson and Péwé (1986), they are simple to program and can be easily implemented on small desktop computers. In the multiple layer model, each layer must also be of constant thickness, although thickness is allowed to vary among layers. Because the new solutions are adapted from the analytic theory of folding, they yield continuous values for stress and displacement throughout the layer, as opposed to the discrete nodal values obtained from finite element solutions. Linear elastic rheology is assumed, so that lithostatic normal stresses can simply be added to the series solutions for perturbed stresses developed as a consequence of draping.
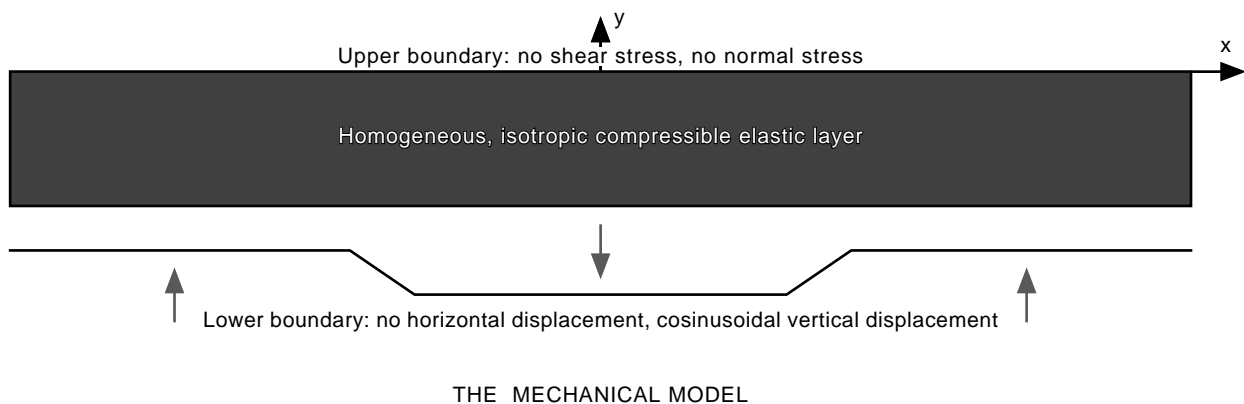


THE  MECHANICAL MODEL

Figure 1. Boundary conditions and geometry of the idealized draping problem. The coordinate system used here is a variation of the system used in Haneberg (1992, 1993), in which the origin is centered over the left-dipping step; this coordinate shift involves only a switch of sine and cosine terms in the published solutions.

Qualitative analysis of the continuum solutions shows that stress and displacement fields developed as a consequence of draping are controlled by: (1) thickness of the layer(s); (2) width of the buried irregularity; (3) amplitude or height of the buried irregularity; (4) stiffness, and to a lesser degree compressibility, of the layer(s); and (5) the shear strength of the lower boundary. The issue of lower boundary shear strength arose during an analysis of tectonic drape folds in sedimentary strata (Haneberg, 1992), and is probably not important in shallow unconsolidated layers.

A series of numerical experiments with the single layer model yields results that may have implications for the origin of earth fissures (figs. 2 and 3). Zones of high stress along the upper boundary correspond to the locations of inflection points along the lower boundary, with tensile stresses developed above convex-upward inflections and compressive stresses developed above concave-upward inflections. If the layer is draped over a broad, low amplitude irregularity, for example a buried channel-fill deposit, tensile stresses are developed only along the upper surface because the confining lithostatic pressure is greater in magnitude than any tension developed at depth. If the irregularity is sufficiently narrow and the layer is sufficiently thin, perhaps corresponding to a buried fault scarp, the model predicts that tensile stresses developed at depth—even in the presence of compressive lithostatic stresses—can be greater than those developed along the ground surface. A narrow step also has the effect of concentrating displacement gradients (i.e., strains) into a narrower zone of larger gradients just above the step.

Because the deposits in which fissures form have little tensile strength, the development of tension at depth means that opening mode cracks may in theory nucleate at the toe of a narrow irregularity and propagate upwards. A propagating opening mode crack will remain perpendicular to the least compressive principal stress in order to maximize the dissipation of strain energy; therefore, an opening mode crack that begins at depth and grows upward will tend to curve away from the irregularity. This phenomenon may explain the location of an earth fissure near San Marcial, New Mexico, that appears to have breached the surface in alluvium above the hanging wall of a small graben-bounding fault (Haneberg and others, 1991).
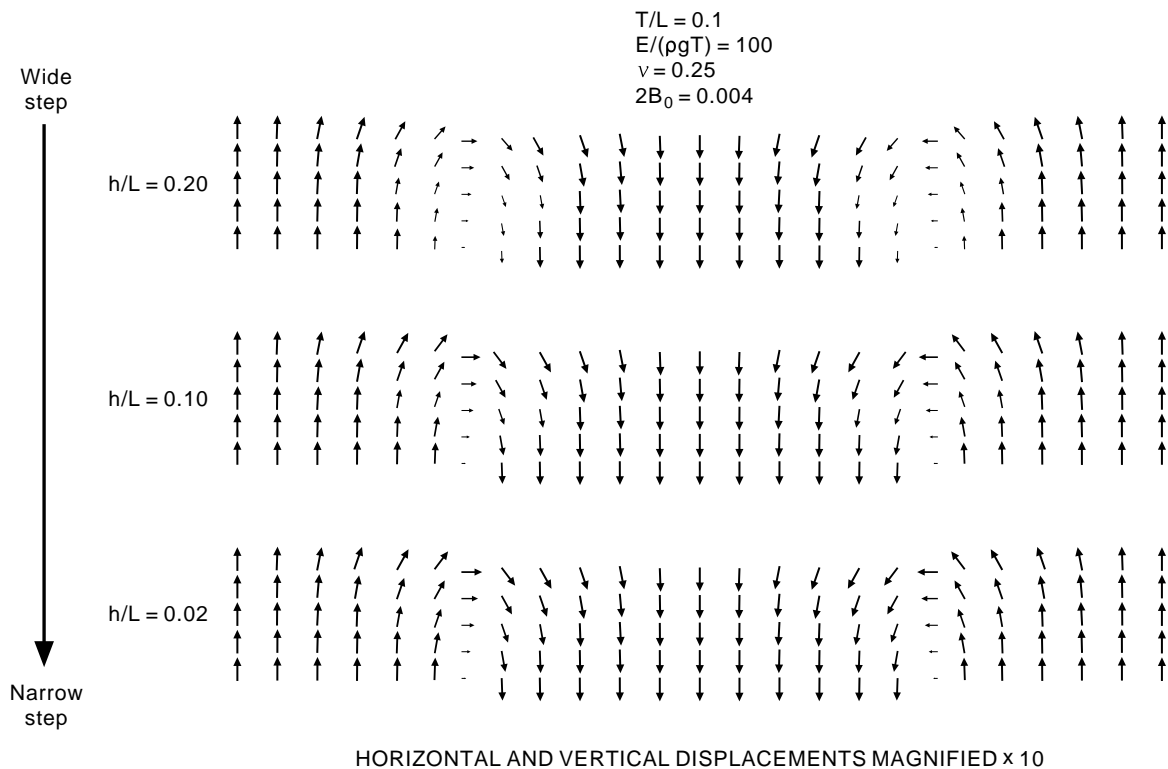
$T/L = 0.1$
$E/(\rho g T) = 100$
$\nu = 0.25$
$2B_0 = 0.004$

Wide
step

h/L = 0.20

h/L = 0.10

h/L = 0.02

Narrow
step

HORIZONTAL AND VERTICAL DISPLACEMENTS MAGNIFIED × 10

Figure 2. Displacement fields produced by draping of a single compressible elastic layer over a pair of facing steps. Variables are: T—layer thickness, L—fold wavelength, E—Young's modulus, g—gravitational acceleration, ν—Poisson's ratio, B—height of step, h—width of step. In order to examine the effects of changing step geometry, the ratio h/L is decreased from 0.20 in the uppermost layer to 0.02 in the lowermost layer, thereby concentrating displacement gradients (i.e., strains) above the steps. In order to emphasize the perturbed stress fields near the steps, gravitational body forces were not included in these calculations.

Bell and others (1992) describe a similar situation, in which fissures are located in the hanging walls of reactivated faults near Las Vegas, Nevada. An opening mode crack that starts at the Earth's surface, in contrast, can be expected to propagate downward only a short distance before compressive lithostatic stresses terminate fracture growth. One might therefore predict that fissures developed along downward propagating cracks will be relatively shallow and quickly filled with sediment, and that fissures developed along upward propagating cracks will be relatively deep and persistent (see Schumann, Morton, Ward and others, Carpenter, and Haneberg and Friesen abstracts for discussions of other earth fissures related to their development). This is not to say that all earth fissures must form along opening mode cracks that originate at depth, particularly because supporting field evidence is weak at best, but rather that the development of tension at depth may be one way to initiate the fissuring process.



$T/L = 0.1$
$E/(\rho g T) = 100$
$\nu = 0.25$
$2B_0 = 0.004$

Wide step

h/L = 0.20

h/L = 0.10

h/L = 0.02

Narrow step

HORIZONTAL AND VERTICAL DISPLACEMENTS MAGNIFIED × 10
COMPRESSION REPRESENTED BY A SOLID SYMBOL,
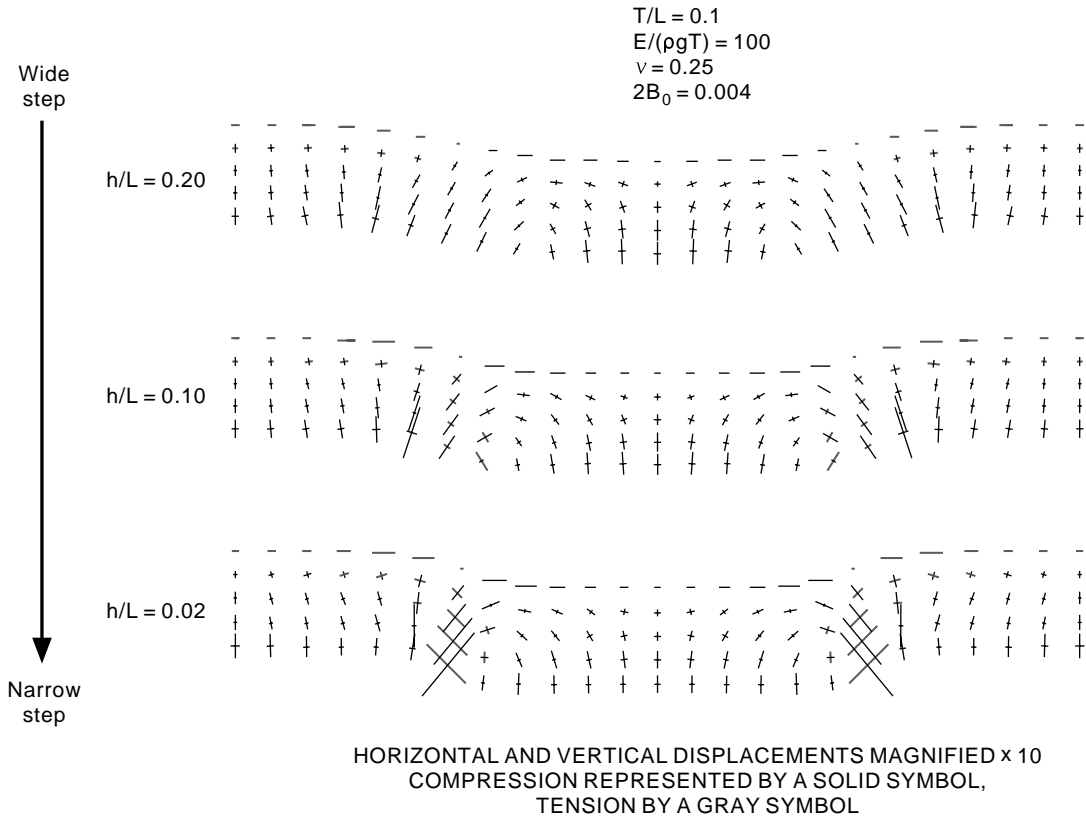TENSION BY A GRAY SYMBOL

Figure 3. Principal stress fields produced by draping of a single compressible elastic layer over a pair of facing steps. Variables are identical to those in figure 2. The magnitude of principal stresses is proportional to length. Tensile stresses, which may initiate fissuring at depth in unconsolidated or poorly consolidated sedimentary aquifers, develop along the step as the ratio h/L is decreased. In order to emphasize the perturbed stress fields near the steps, gravitational body forces were not included in these calculations.